Physics 201 36 Name KEY

Exam 2 – Capacitance, Circuits and Magnetism

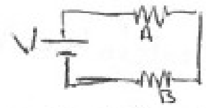
March 5, 2009

This is a closed book examination. However, you may use a 8.5" x 11" sheet of paper with your own notes during this exam. There is extra scratch paper available. Please explain your answers. Your explanation is worth 3/4 of the points on multiple-choice questions.

1) [4 PTS] What is the net force on a neutron ($q = 0$) with velocity $\vec{v} = 50 m/s\ \hat{i}$ traveling through a region of space with $\vec{E} = 40 V/m\ \hat{j}$ and $\vec{B} = 1.5T\ \hat{k}$?

$\vec{F} = q(\vec{E} + \vec{v} \times \vec{B})$ $q = 0$ $\therefore$ $\vec{F}_{NET} = 0$

2) [4 PTS] Two light bulbs (A and B) are connected in series. Bulb A is twice as bright as bulb B. What must be true?



$P_A = 2P_B$ $I_A = I_B$

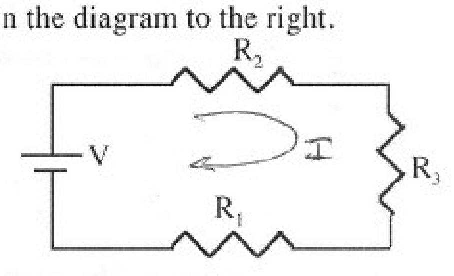
a) $2R_B=R_A$
b) $\sqrt{2}\,I_B=I_A$
c) Bulb A receives the current from the battery first (closest to positive terminal)
d) Bulb B receives the current from the battery first (closest to positive terminal)
e) $2R_A=R_B$
f) $\sqrt{2}\,I_A=I_B$

$P = IV = I^2R$ $V = IR$

$I_A^2 R_A = 2 I_B^2 R_B$

$I^2 R_A = 2 I^2 R_B$ $\therefore$ $R_A = 2R_B$

3) [4 PTS] You connect three resistors to a battery as shown in the diagram to the right. Which resistor has the most current flowing through it?



a) R_1
b) R_2
c) R_3
d) Which ever one has the smallest resistance.
e) Which ever one has the largest resistance.
f) The current is the same through all of them.

Current is constant through objects in series –
* current in is equal to current out

4) [4 PTS] You are measuring the voltage across a capacitor with a charge Q on it. How does the energy change when you insert a dielectric with K=2 into the capacitor.

a) The energy decreases.
b) The energy does not change.
c) The energy increases.

$C = \frac{Q}{V}$ $V = \frac{Q}{C}$ $C_0 = \frac{\epsilon_0 A}{d}$

$E = \frac{\sigma}{\epsilon}\hat{x} = \frac{Q}{\epsilon A}\hat{x}$

$V = \int \vec{E} \cdot d\vec{x} = \frac{Q}{\epsilon A} d$

$C = \frac{Q}{V} = \frac{Q}{Qd/\epsilon A} = \frac{\epsilon A}{d}$

$U = \frac{1}{2}QV = \frac{1}{2}\frac{Q^2}{C}$

$U_0 = \frac{1}{2}\frac{Q^2}{\epsilon_0 A} d$

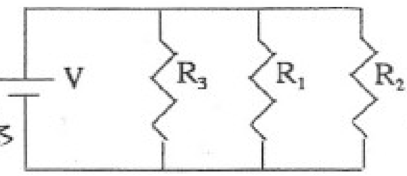
$U_1 = \frac{1}{2}\frac{Q^2}{K\epsilon_0 A} d$

Charge is constant
Capacitance will change

$\therefore U_1 = \frac{1}{K} U_0$

Voltage changes

5) [4 PTS] The light bulbs in the circuit to the right have different resistances, $R_1 = 2R_2 = 4R_3$. Which bulb is brightest (uses the most power)?



a) R_1
b) R_2
c) R_3
d) All the light bulbs are of equal brightness since they have the same voltage across them.

Voltage is same across objects in parallel

$P = IV = V^2/R$

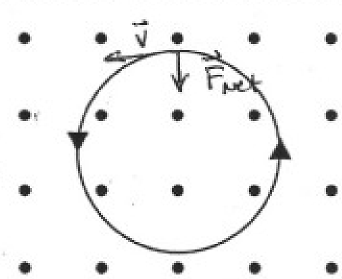
∴ Bulb with lowest R has greatest power – $R_3 < R_2 < R_1$

6) [4 PTS] A charged particle is moving in a uniform magnetic field (coming out of the page) as shown in the figure to the right. What type of particle would follow the path indicated?

a) Proton
b) Neutron
c) Electron
d) Photon

$\vec{F} = q\vec{v} \times \vec{B}$

For circular motion you need a central force (acceleration)



$\vec{v} \times \vec{B}$ gives a force radially outward... – so charge needs to be negative

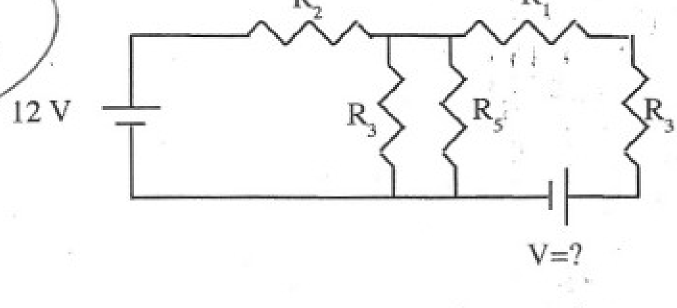
Note: A positive charge would go in a CW direction

7) [12 PTS] A charged capacitor (C=3.2 mF) is connected to a resistive load (R=100 Ω) at time t=0s. The capacitor is initially charged to 5.1 Volts.

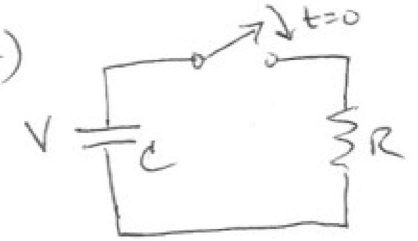
a) What is the time constant for this circuit? $\tau = RC = (100\Omega)(3.2\text{mF}) \approx 0.32\text{s}$
b) What is the initial energy stored in the capacitor? $U = \frac{1}{2}QV = \frac{1}{2}(CV)V = \frac{1}{2}CV^2 = \frac{1}{2}(3.2\text{mF})(5.1\text{V})^2$
c) How much energy is left in the capacitor at time t=τ?
d) What is the power used by the load as a function of time?

8) ~~[12 PTS]~~ Solve for the unknown source voltage, the current through resistor R_1 and the power used by resistor R_5 in the diagram below. The current through R_2 is 1,666 mA while $R_1 = 8\ \Omega$, $R_2 = R_5 = 2\ \Omega$ and $R_3 = 4\ \Omega$.

NOT INCLUDED



(7)



$V_0 = 5.1\,V$

$C = 3.2\,mF$

$R = 100\,\Omega$

(a) $\tau = RC = (100\,\Omega)(3.2\,mF) = 0.32\,s$

(b) $U_0 = \frac{1}{2} QV = \frac{1}{2}(CV)V = \frac{1}{2} CV^2 = \frac{1}{2}(3.2\,mF)(5.1)^2 = 0.042\,J$ (42 mJ)

(c) The voltage (and current) follow an exponential decay

$$V(t) = V_0 e^{-t/\tau} \qquad I(t) = C \frac{d}{dt}V(t) = CV_0 \frac{d}{dt} e^{-t/\tau} = CV_0\left(-\frac{1}{\tau}\right) e^{-t/\tau}$$

$$I(t) = -\frac{V_0}{R} e^{-t/\tau}$$

$$P(t) = V(t)\, I(t)$$
$$= -\frac{V_0^2}{R} e^{-t/\tau} \cdot e^{-t/\tau} = \frac{-V_0^2}{R} e^{-2t/\tau}$$

Power is negative since energy is leaving –

$$U(t) = \int_0^t P(t)\,dt = -\frac{V_0^2}{R}\int e^{-2t/\tau}\,dt = +\frac{\tau}{2}\frac{V_0^2}{R} e^{-2t/\tau} + \text{Constant} = \frac{1}{2}CV_0^2 e^{-2t/\tau} + \text{Const.}$$

Find constant by noting $U(0) = \frac{1}{2} C V_0^2$ [from (b) above]

$\therefore$ constant is $\emptyset$

$$U(t) = \frac{1}{2} CV_0^2 e^{-2t/\tau} \qquad t = \tau$$

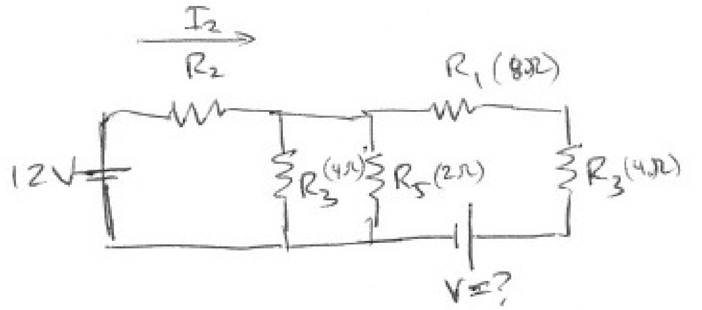
$$U(\tau) = \frac{1}{2} CV_0^2 e^{-2}$$ so at time $t = \tau$ $\quad U(\tau) = \frac{U_0}{e^2} = 0.0057\,J$ (5.7 mJ)

(d) From above

$$P(t) = -\frac{V_0^2}{R} e^{-2t/\tau}$$ leaving capacitor so $P(t)$ ~~from~~ used by load is (–)

LOAD
$$P(t) = \frac{V_0^2}{R} e^{-2t/\tau} = 0.26\, e^{-2t/\tau}\ W$$
$$= 0.26\, e^{-t/1.6}\ W$$

(8)



$I_2 = 1.666\text{A}$

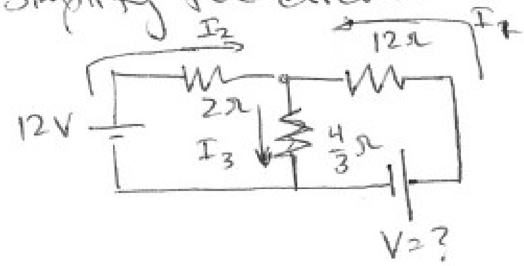
$R_1 = 8\,\Omega$

$R_2 = R_5 = 2\,\Omega$

$R_3 = 4\,\Omega$

NOTE $R_5 // R_3 \Rightarrow \left[\frac{1}{R_5} + \frac{1}{R_3}\right]^{-1} = \left(\frac{1}{4} + \frac{1}{2}\right)^{-1} = \frac{4}{3}\,\Omega$

Simplify the circuit



(A) $I_2 + I_1 = I_3$ current in equals current out

$\Delta V = 0$ voltage around loop is zero

(B) $12\text{V} - I_2(2\Omega) - I_3(4/3\,\Omega) = 0$

(C) $V - I_1(12\Omega) - I_3(4/3\,\Omega) = 0$

FROM (B) $\dfrac{12\text{V} - (1.666\text{A})(2\Omega)}{4/3\,\Omega} = I_3 = \frac{13}{2}\text{A}$ (6.5 A)

FROM (A) $I_1 = I_3 - I_2 = \frac{13}{2}\text{A} - \frac{5}{3}\text{A} = \frac{29}{6}\text{A}$ (4.83 A)

FROM (C) $V = (12\Omega)(\frac{29}{6}\text{A}) + (\frac{4}{3}\Omega)(\frac{13}{2}\text{A}) = 58\text{V} + \frac{26}{3}\text{V} = 66.6\text{V}$

Power through R_5 $P_{R5} = IV = \dfrac{V^2}{R}$

$P_{R5} = (26/3\,\text{V})^2 / 2\Omega = 37.6\text{ W}$

use voltage since current through $R_3 // R_5$ is not same but voltage is the same

$V = (\frac{4}{3}\Omega)(\frac{13}{2}\text{A}) = \frac{26}{3}\text{V}$